



**FEDERAL PUBLIC SERVICE COMMISSION**  
**COMPETITIVE EXAMINATION-2018**  
**FOR RECRUITMENT TO POSTS IN BS-17**  
**UNDER THE FEDERAL GOVERNMENT**  
**PURE MATHEMATICS**

Roll Number

**TIME ALLOWED: THREE HOURS**

**MAXIMUM MARKS = 100**

- NOTE: (i)** Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C**. **ALL** questions carry **EQUAL** marks.
- (ii)** All the parts (if any) of each Question must be attempted at one place instead of at different places.
- (iii)** Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
- (iv)** No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
- (v)** Extra attempt of any question or any part of the attempted question will not be considered.
- (vi)** **Use of Calculator is allowed.**

**SECTION-A**

- Q. 1.** (a) Let  $H$  and  $K$  be normal subgroups of a group  $G$ . Show that  $HK$  is a normal subgroup of  $G$ . (10)
- (b) Let  $H$  and  $K$  be normal subgroups of a group  $G$  such that  $H \subseteq K$ . Then show that (10) (20)
- $$(G/H)/(K/H) \cong G/K$$
- Q. 2.** (a) Show that every finite integral domain is a field. (10)
- (b) Consider the following linear system, (10) (20)
- $$\begin{aligned}x + 2y + z &= 3 \\ay + 5z &= 10 \\2x + 7y + az &= b\end{aligned}$$
- (i) Find the values of  $a$  for which the system has unique solution.
- (ii) Find the values of the pair  $(a, b)$  for which the system has more than one solution.
- Q. 3.** (a) Find condition on  $a, b, c$  so that vector  $(a, b, c)$  in  $\mathbb{R}^3$  belongs to (10)
- $$W = \text{span} \{u_1, u_2, u_3\}$$
- where
- $u_1 = (1, 2, 0)$
- ,
- $u_2 = (-1, 1, 2)$
- ,
- $u_3 = (3, 0, -4)$
- . (10) (20)
- (b) Let  $W_1$  and  $W_2$  be finite dimensional subspaces of a vector space  $V$ . Show that
- $$\dim W_1 + \dim W_2 = \dim (W_1 \cap W_2) + \dim (W_1 + W_2)$$

**SECTION-B**

- Q. 4.** (a) Let  $f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ x & \text{if } x > 1 \end{cases}$  (10)
- Does the Mean Value Theorem hold for  $f$  on  $\left[\frac{1}{2}, 2\right]$ .
- (b) Calculate the.  $\lim_{x \rightarrow 0} \frac{\ln \sin 3x}{\ln \sin x}$  (10) (20)
- Q. 5.** (a) Evaluate  $\int_{-1}^5 |x-2| dx$ . (10)
- (b) Prove that  $f_{xy}(0,0) \neq f_{yx}(0,0)$  if (10) (20)
- $$f(x, y) = \begin{cases} x^2 y \sin \frac{1}{x} & \text{when } x, y \text{ are not both } 0 \\ 0 & \text{when } x, y \text{ are both } 0 \end{cases}$$

## PURE MATHEMATICS

- Q. 6. (a)** Find the area of the region bounded by the cycloid  $x = a(\theta - \sin \theta)$ ,  $y = a(1 - \cos \theta)$  and its base. (10)
- (b)** Find the equation of a plane through (5,-1,4) and perpendicular to each of the planes  $x + y - 2z - 3 = 0$  and  $2x - 3y + z = 0$  (10) (20)

### SECTION-C

- Q. 7. (a)** Express  $\cos^5 \theta \sin^3 \theta$  in a series of sines of multiples of  $\theta$ . (10)
- (b)** Use Cauchy's Residue Theorem to evaluate the integral  $\int_C \frac{5z-2}{Z(Z-1)} dz$  where  $C$  (10) (20)  
is the circle  $|z|=2$ , described counter clock wise.
- Q. 8. (a)** Find the Laurent series that represent the function  $f(z) = \frac{z+1}{z-1}$  in the domain  $1 < |z| < \infty$ . (10)
- (b)** Expand  $f(x) = \sin x$  in a Fourier cosine series in the interval  $0 \leq x \leq \pi$ . (10) (20)

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