



**FEDERAL PUBLIC SERVICE COMMISSION  
COMPETITIVE EXAMINATION-2021  
FOR RECRUITMENT TO POSTS IN BS-17  
UNDER THE FEDERAL GOVERNMENT**

Roll Number

**PURE MATHEMATICS**

<b>TIME ALLOWED: THREE HOURS</b>	<b>MAXIMUM MARKS = 100</b>
<p><b>NOTE: (i)</b> Attempt <b>FIVE</b> questions in all by selecting <b>TWO</b> Questions each from <b>SECTION-A&amp;B</b> and <b>ONE</b> Question from <b>SECTION-C</b>. <b>ALL</b> questions carry <b>EQUAL</b> marks.</p> <p><b>(ii)</b> All the parts (if any) of each Question must be attempted at one place instead of at different places.</p> <p><b>(iii)</b> Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.</p> <p><b>(iv)</b> No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.</p> <p><b>(v)</b> Extra attempt of any question or any part of the attempted question will not be considered.</p> <p><b>(vi)</b> <b>Use of Calculator is allowed.</b></p>	

**SECTION-A**

- Q. 1. (a)** Let  $\Psi$  be a homomorphism of group  $G$  into group  $\tilde{G}$  with kernel  $K$ , prove that  $K$  is a normal subgroup of  $G$ . (10)
- (b)** Prove that if  $H$  and  $K$  are two subgroups of a group  $G$ , then  $HK$  is a subgroup of  $G$  if and only if  $HK=KH$ . (10) (20)

- Q. 2. (a)** Find elements of the cyclic group generated by the permutation. (10)

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 2 & 6 & 1 \end{pmatrix}$$

- (b)** Verify that the polynomials  $2-x^2$ ,  $x^3-x$ ,  $2-3x^2$  and  $3-x^3$  form a basis for the set  $P_3(x)$ ; the set of all polynomials of degree three. Also express the vectors  $1+x^2$  and  $x+x^3$  as a linear combination of these basis vectors. (10) (20)

- Q. 3. (a)** Let  $V$  be the real vector space of all function from  $R$  to  $R$ . Show that  $\{\cos^2 x, \sin^2 x, \cos 2x\}$  is linearly dependent while  $\{\cos x, \sin x, \cosh x, \sinh x\}$  are linearly independent. (10)

- (b)** Solve the system of linear equations: (10) (20)

$$\begin{aligned} x_1 - 2x_2 - 7x_3 + 7x_4 &= 5 \\ -x_1 + 2x_2 + 8x_3 - 5x_4 &= -7 \\ 3x_1 - 4x_2 - 17x_3 + 13x_4 &= 14 \\ 2x_1 - 2x_2 + 11x_3 + 8x_4 &= 7 \end{aligned}$$

**SECTION-B**

- Q. 4. (a)** If  $f(x, y) = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$ . (10)

Show that  $\frac{\partial^2 f}{\partial y \partial x}(x, y) = \left(\frac{x^2 - y^2}{x^2 + y^2}\right)$

- (b)** Evaluate  $\int_0^6 f(x) dx$  where  $f(x) = \begin{cases} x^2 & \text{when } x < 2 \\ 3x - 2 & \text{when } x > 2 \end{cases}$  (10) (20)

## PURE MATHEMATICS

**Q. 5. (a)** Let  $I_n = \int_0^{\infty} x^n e^{-x} dx$  where  $n$  is an integer. Prove that (10)  
 $I_n = n I_{n-1}$  Hence show that  $I_n = n!$

**(b) i.** Write  $r = \frac{8}{2 - \cos \theta}$  in rectangular coordinates. (10) (20)

**ii.** Write  $x^4 + 2x^2y^2 + y^4 - 6x^2y + 2y^3 = 0$   
in polar coordinates.

**Q. 6. (a)** Evaluate  $\iint_D dydx$  and  $\iint_D dx dy$  where  $D$  is the region bounded by the  $y$ -axis, the (10)  
lines  $x=2$  and the curve  $e^x$ .

**(b)** Investigate the curve  $y = \frac{x^3 - x}{3x^2 + 1}$  for points of inflexion. (10) (20)

### SECTION-C

**Q. 7. (a)** Sum the series  $1 + \frac{1}{2} \cos \theta + \frac{1.3}{2.4} \cos 2\theta + \frac{1.3.5}{2.4.6} \cos 3\theta + \dots$  (10)

**(b)** Prove that  $\cos \frac{\pi}{7} - \cos \frac{2\pi}{7} + \cos \frac{3\pi}{7} = \frac{1}{2}$  (10) (20)

**Q. 8. (a)** Construct the analytic function  $f$  whose real part is  $U = x^3 - 3xy^2 + 3x + 1$  (10)

**(b)** Evaluate  $\int_C \frac{dz}{z^2 + 2z + 2}$  Where  $C$  is a square with corners (10) (20)  
(0,0),(-2,0),(-2,-2) and (0,-2).

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