FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2023 FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS = 100**
NOTE: (i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks. **(ii)** All the parts (if any) of each Question must be attempted at one place instead of at different places. **(iii)** Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper. **(iv)** No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed. **(v)** Extra attempt of any question or any part of the attempted question will not be considered. **(vi)** **Use of Calculator is allowed.**	

SECTION-A

Q. 1. **(a)** Find centre of S_3. (10)

(b) Using the row operations, show that the matrix $\begin{pmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{pmatrix}$ has no inverse. (10) **(20)**

Q. 2. **(a)** For any group G, show that $\frac{G}{\{e\}} \cong G$ and $\frac{G}{G} \cong \{e\}$. (10)

(b) Suppose U and W are distinct four dimensional subspaces of a vector space V of dimension six. Find the possible dimension of $U \cap W$. (10) **(20)**

Q. 3. **(a)** For what value of α is the matrix $\begin{pmatrix} -\alpha & \alpha-1 & \alpha+1 \\ 1 & 2 & 3 \\ 2-\alpha & \alpha+3 & \alpha+7 \end{pmatrix}$ is singular? (10)

(b) Define $T:\Re^3 \to \Re^3$ by $T(x_1,x_2,x_3)=(-x_3,x_1,x_1+x_3)$. Find $N(T)$. Is T one-to-one? (10) **(20)**

SECTION-B

Q. 4. **(a)** Find the value of θ and the limit in order that $\lim_{x\to 0}\frac{\sin 2x+\theta\sin x}{x^3}$ be finite. (10)

(b) Show that $x<\sin^{-1}x<\frac{x}{\sqrt{1-x^2}}$, 0<x<1. (10) **(20)**

Q. 5. **(a)** Given that $U=\frac{1}{\sqrt{x^2+y^2+z^2}}$. Verify that $U_{xx}+U_{yy}+U_{zz}=0$. (10)

(b) Evaluate $\iint (x^2+y^2)dxdy$, over the domain bounded by $y=x^2$ and $x=y^2$. (10) **(20)**

Q. 6. **(a)** Evaluate $\iint (x^2+y^2)dxdy$, over the region bounded by xy=1, y=0, y=x and x=2. (10)

(b) Find an equation of a normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ in the form $ax\cos\theta+by\cot\theta=a^2+b^2$. Prove that the normal is external bisector of the angle between the focal distances of its foot. (10) **(20)**

SECTION-C

Q. 7. **(a)** Determine k such that $U = e^{2x}\cos ky$ is harmonic and find a conjugate harmonic. (10)

(b) Evaluate $\int_C (\frac{1}{z^5} + z^3)dz$ from 1 to -1 along the upper arc of the unit circle. (10) **(20)**

Q. 8. **(a)** Find the Laurent Series of $\frac{1}{1-z^2}$ in the region $0<|z-1|<2$. (10)

(b) Find the residues at the singular points of $\frac{-Z^2-22z+8}{Z^3-5z^2+4z}$ which lie inside the circle $|z|=2$. (10) **(20)**

****