FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2026 FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT

PURE MATHEMATICS

Roll Number

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS = 100**

NOTE:
(i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

SECTION-A

Q. No.1.(a) State the Lagrange's Theorem and find all the subgroups of the cyclic group of order 24. Show that any group of prime order has no non-trivial subgroups. **(10)**

(b) Let G be an Abelian group and H is a subset of G consisting of those elements of G which are of the second order then H is a subgroup of G. Let $G = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{R}, ad - bc \neq 0\right\}$ be the group of all matrices under matrix multiplication. Show that $H = \left\{\begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in \mathbb{R}, ad \neq 0\right\}$ is a subgroup of G and $K = \left\{\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} : b \in \mathbb{R}\right\}$ is a subgroup of H. **(10)**

Q. No.2. (a) Consider the group $\mathbb{Z}_6$ under addition modulo 6. Let $\psi(x) = 5x(mod\,6)$. Verify that it is an automorphism. Define Kernel and Image with at least one example of each. Consider the homomorphism: $\psi: \mathbb{Z}_4 \to \mathbb{Z}_2$, where $\psi(x) = x(mod\,2)$. Find $Ker(\psi)$? Let $\Phi: G \to H$ be a group homomorphism, and let $g \in G$. Then for any integer n, we have $\Phi(g^n) = \Phi(g)^n$. **(10)**

(b) A computer chip manufacturer decides to spend 6×10^5 rupees on radio, magazine, and TV advertisement. If he spends as much on TV advertisement as on magazine and radio together and the amount spent on magazine and TV combined equals five times that spent on radio. What is the amount to be spent on each type of advertisement? Use Gauss Jordan Method. **(10)**

Q. No.3. (a) Find Echelon form and Rank of the following matrix by successively reducing it to its row/column sub-matrices. **(10)**

$$\begin{bmatrix} 1 & -2 & 1 & 0 & 5 \\ -1 & 0 & 1 & -2 & 2 \\ 2 & -3 & 0 & 2 & 3 \end{bmatrix}$$

(b) Define linearly independent and linearly dependent vectors. Are the vectors *(1, −2, 4, 1), (2, 1, 0, −3) and (1, −6, 1, 4)* in R^4 linearly independent or linearly dependent? **(10)**

SECTION-B

Q. No.4. (a) Consider two planes as follows: **(10)**

$$P_1: 2x - y - 2z + 5 = 0,$$
$$P_2: 4x - 2y - 4z + 15 = 0.$$

Are the planes parallel? If no, what is angle between them?

(b) Derive the equation of sphere in standard form. Find the equation of sphere through the circle: **(10)**

$$x^2 + y^2 + z^2 = 9,\ 2x + 4y + 5z = 6 \text{ and touching the } xz\text{-plane.}$$

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Q. No.5. (a) Find cylindrical coordinates of the point with rectangular coordinates $(2\sqrt{3}, 2, -2)$. Express the equation $(x+y)^2 - z^2 + 4 = 0$ in cylindrical and spherical coordinates. **(10)**

(b) Use Maclaurin Series to prove that

$tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots,$ hence find the value of π. **(10)**

Q. No.6. (a) Evaluate the following limits: **(10)**

$$(i) \lim_{x \to 0} \left[\frac{1}{\sin 3x} - \frac{1}{3x}\right] \qquad (ii) \lim_{x \to 0} \frac{x - \sin x}{x^3}$$

(b) What is the area A, of the region bounded by the curves $y = x^2$ and $y = \sqrt{x}$? **(10)**

SECTION-C

Q. No.7. (a) State De-Moivre's Theorem. If $x + \frac{1}{x} = 2\cos\theta$, $y + \frac{1}{y} = 2\cos\varphi$, $z + \frac{1}{z} = 2\cos\phi$, prove that $xyz + (xyz)^{-1} = 2\cos(\theta + \varphi + \phi)$. **(10)**

(b) State Green's Theorem and verify it for $\oint_C (2xy - x^2)dx + (x + y^2)dy$, where C is the curve bounded by $y = x^2$ and $x = y^2$. **(10)**

Q. No.8. (a) Use the concept of Path Integrals to prove following: **(10)**

$(i)\ \oint_{|z|=1} \frac{1}{z}\, dz = 2\pi i \qquad (ii)\ \oint_{|z|=1} z\,dz = 0$

(b) State Residue Theorem and use it to prove the following: **(10)**

$\oint_C \frac{e^{-z}}{(z+2)^5}\, dz = \frac{1}{12}\pi i e^2$ where C is the circle $|z| = 3$.

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